Distance decay function and its application in the territory of the Czech Republic

General subject of geographical research, the planet Earth, has distinctly non-homogenous features. The non-homogeneity brings about polarity that tends to be naturally levelled and thus the geography becomes the discipline of distance to the considerable extent (Haggett 1965). The natural levelling of the polarity holds almost completely true for physical geographic processes, however, some significant human geographical phenomena behave rather similarly. The levelling of existing polarities on Earth demands for some kinds of horizontal flows. They take on different forms as the flows of material, energy, information, individuals, etc., and significantly influence the geographic organisation of space expressing a mutual relation between sections of geographic space.

It is quite obvious that these flows occur along some distance in space. The role of distance in geographical research has started to be ascertained since the end of the 19th century in the works of Ravenstein (e.g. 1885) on migration in Great Britain, though the effect of distance on the migration patterns and intensities had not been fully recognised. However, these pioneering studies have given rise to the modelling of spatial interactions and thus have attracted the heightened attention to the influence of distance on spatial patterns and interaction intensities.

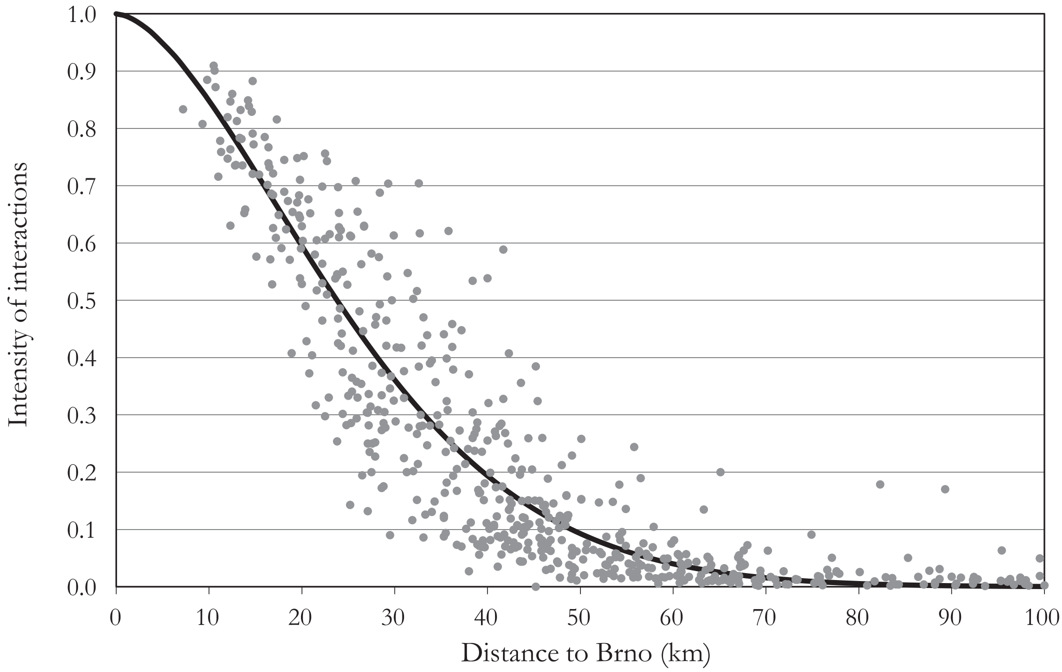
During the interwar period Reilly (1929, 1931) has discussed a decrease of intensity in shopping trips with distance, formulating the Reilly's law of retail gravitation inspired by the laws of physics. In the 1940s Stewart (1948) coined a term "social physics" and claimed that the physical laws do not have to be obeyed when studying the behaviour of individuals. His conclusions were to a certain extent influenced by the principle of least effort (Zipf 1947). However, the principles of Newtonian physics were not questioned, only the frictional effect of distance did not apply the square root necessarily.

Tobler's (1970:236) first rule of geography states that "everything is related to everything else but near things are more related than distant things." But here the question has arisen then, what shape or form does the decrease in the interaction intensity along the distance take on. Physical analogy of inverse square root function or just simple inverse function applied for instance by Isard (1960) or Haggett (1965) have been questioned later for instance by Wilson (1974), Alonso (1978) or Fotheringham, O'Kelly (1989) as either just for the physical analogy applied on society and individuals or for their simplicity as they have rather insufficiently approximated the reality.

Goux (1962) has suggested several functions that are able to describe the distance decay in interaction intensities more appropriately and Wilson (1971) called them a family of distance decay functions. The issue has been further developed and discussed for instance by Taylor (1971, 1983). The correct shape of the distance decay function is crucial for spatial interaction modelling and the calibration of models applied in various geographical milieus including migration, labour commuting, transport, retail behaviour, regional taxonomy, etc.

According to data transformation Taylor (1971) identifies models of functions with untransformed data, single-log models and double-log models. According to shape and course of distance decay power, exponential, log-normal and Pareto functions can be identified. The combination of power and exponential functions produces so called Tanner function. Specific cases of distance decay functions can be identified with spatial monopolies when the choice of a particular phenomenon or location is not possible or allowed. A situation when 100 per cent of the population of an area have to travel to a particular centre in order to fulfil some administrative acts is the example. Population of another area similarly travels to another centre, then.

When plotting the distance decay without the data transformation it is the most suitable to use a bell shaped function with the inflexion point, which asymptotically approaches zero with the increasing distance. Examples of such functions are less numerous in the world scientific literature. However, we should mention herein Box-Cox function



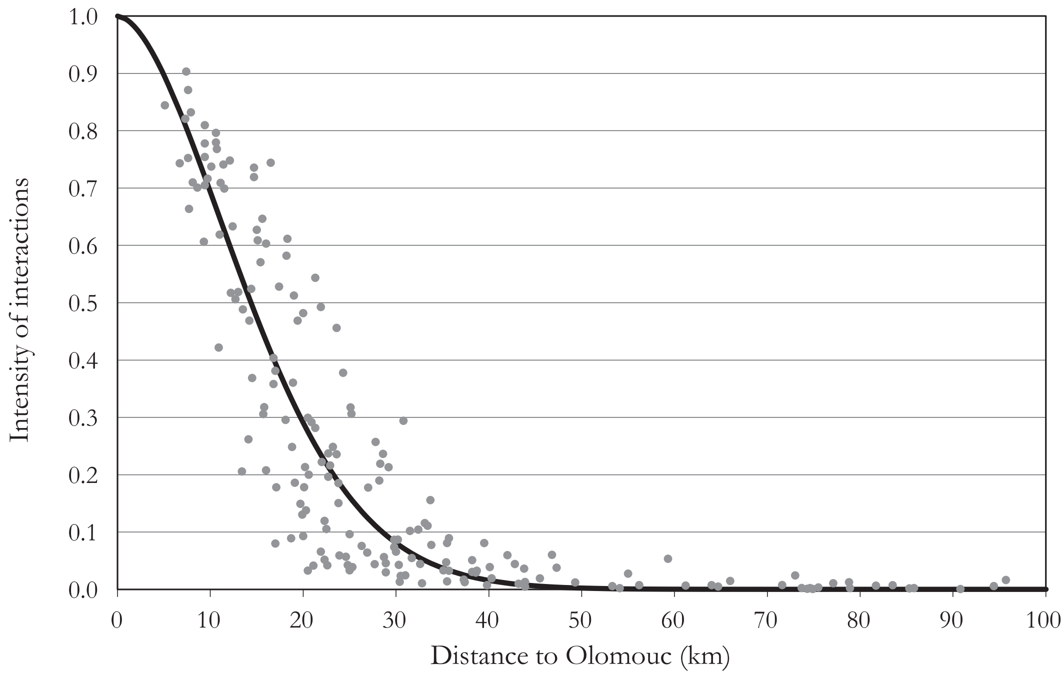


Figure 1 Distance decay functions for the daily travel-to-work-flows to Brno and Olomouc.
Source: own computation.

(Ortúraz, Willusen 2001) and Richards function with no fewer than four parameters (Martínez, Viegas 2013), which were applied for the first time in botany (Richards 1959). As the most suitable this function has been also applied in the modelling of distance decay function for the daily travel-to-work flows to the regional centres of the Czech Republic. The function has been determined the daily travel-to-work flows to all centres with the population higher than 25,000. It was the power-exponential function of the form:

$$f(d) = \exp(-\alpha \cdot d^{\beta}) \qquad [1]$$

where d is a distance from a centre; α, β are parameters, $\alpha > 0$, $\beta > 0$. For each centre an individual function has been constructed with the most fitting parameters α, β (Figure 1 presents the functions for the workplaces of both authors of this research report – Brno and Olomouc). The application of statistical methods produced the most suitable general value of the β parameter, i.e., 1.57. The use of the constant parameter β with the value 1.57 brings about only an insignificant decrease of the coefficient of determination in comparison to the employment of optimal values of this parameter.

It is yet possible, with further decrease of the coefficient of determination, to express the general (unified) distance decay function for a particular centre based on the number of jobs or even on the number of inhabitants (which is regarded as the most general quantitative characteristic). The calculation of the area lying below the function in the graph enables us to determine the range of the influence radius of particular centres. Complete results of analyses of the distance decay functions for daily travel-to-work flows to the regional centres of the Czech Republic have been processed so far and will be submitted to publication in the prestigious scientific journal.

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